References

[1]Frost, W. and Bowles, R. L., "Wind Shear Tenure in the Equations of Aircraft Motion," *Journal of Aircraft*, Vol. 21, Nov. 1984, pp. 866–872.

[2]Etkin, B., "Turbulent Wind and Its Effect on Flight," *Journal of Aircraft*, Vol. 18, May 1981.

[3]Etkin, B., *Dynamics of Atmospheric Flight*, Wiley, New York, 1972, pp. 327–345.

Comment on "Body of Revolution Comparisons for Axial- and Surface-Singularity Distributions"

John M. Kuhlman*
West Virginia University, Morgantown, West Virginia

In a recent Note,[1] D'Sa and Dalton present two comparisons of numerical potential flow surface velocity results for a surface source and an axial source singularity method. This work presents useful comparisons between the two techniques and points out the likelihood that axial-singularity methods may yield inaccurate solutions with arbitrary choices of size distribution of the axial-singularity elements. This dependence of solution accuracy on the size distribution of axial-singularity elements appears to be stronger for the higher-order formulations (e.g., quadratic), as seen in Fig. 2 of Ref. 1. Certainly, the general conclusion that surface-singularity methods are as a result more reliable than axial-singularity methods is valid. However, some limitations to this work must be pointed out. These limitations occur in two general areas: the limited usefulness of the ellipsoidal body as a test case for axial-singularity methods, as recently pointed out by Hess,[2] and what appears to be an inadequate, or less than optimal, formulation for the axial-singularity method, especially as applied to the second test geometry of Ref. 1.

First, as Hess reminds us in Ref. 2, the axisymmetric potential flow past an ellipsoid may be *exactly* represented by a line source distribution of linearly varying intensity, located between the foci, independent of body slenderness ratio. Hence, it is most likely that axial line source singularity numerical results for assumed piecewise linear source intensity will be a measure of the influence of finite word length and accumulated truncation error in the calculation, assuming that the proper inset of the axial source distribution is used.

Also, based on the presented results for the axial source distributions for the ellipsoid (Fig. 1, Ref. 1), it appears likely that these calculated results were obtained for a nonoptimal inset of the axial source distribution. Similar small oscillating speed or pressure coefficient error distributions were seen by Kuhlman and Shu[3,4] for ellipsoids at zero angle of attack, using their axial-singularity formulation if nonoptimal inset was used. As found empirically,[3,4] and as shown by Moran,[5] this optimal inset distance is the distance from the near focus to the nose of the ellipsoid. Using the optimal inset, calculated errors in surface pressure coefficient for ellipsoids were found in Ref. 3 to be $0(10^{-5})$ using 60-bit word length, a flow tangency boundary condition formulation, and 15 linearly varying line source elements in a cosine size distribution to represent each half of the body.

Received Oct. 22, 1986; revision submitted Nov. 3, 1986.

*Visiting Associate Professor, Mechanical and Aerospace Engineering Department. Member AIAA.

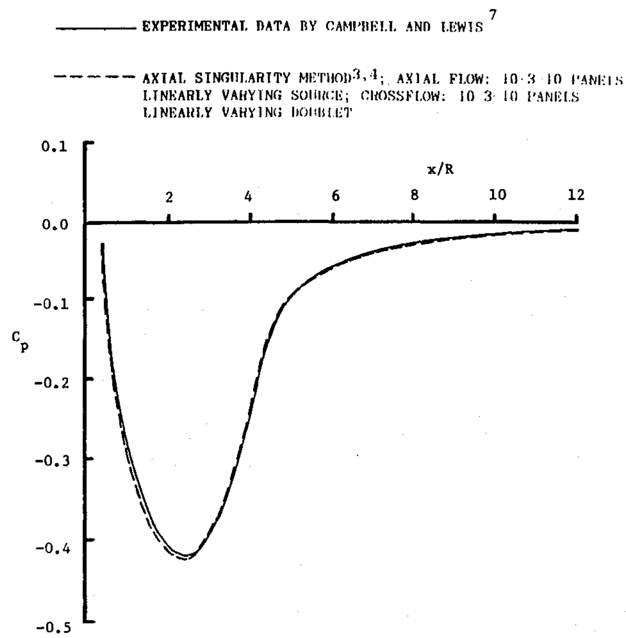

Fig. 1 Pressure coefficient distribution along top meridian line for inclined flow; $\alpha = 6.08$ deg past a cylindrical body with $SR = 2$ ellipsoidal nose, using axial-singularity method, compared with experiment.

Further, using this inset for a slenderness ratio 5 ellipsoid at zero angle of attack, results presented in Ref. 3 showed a continuous trend of decreasing rms error as the number of singularity elements was varied from 10 to 40 for either constant or linearly varying axial source or doublet singularities, using either equal-sized singularity elements or a cosine-type size distribution. This consistent decrease in rms error in calculated surface pressure coefficient distribution contrasts with results presented by D'Sa and Dalton, in which surface velocity error was observed to begin to increase for the ellipsoid test case when more than 35 linear source elements were used. This must be the result of differences in accumulated errors due to finite word length, where the results of Refs. 3 and 4 were obtained using single-precision calculations on CDC hardware, which is roughly equivalent to using double-precision arithmetic on many other computers.

The work of Kuhlman and Shu[3,4] documented the extension of axial-singularity methods to potential flows past axisymmetric bodies at nonzero angle of attack, as suggested by Karamcheti.[6] Accuracies at nonzero angle of attack were identical to those at zero angle of attack, and accuracy was independent of slenderness ratio. It appears likely that a superposition of the piecewise linearly varying cross-flow line-doublet singularity distribution of Kuhlman and Shu[3,4] with the axisymmetric surface-singularity method described by D'Sa and Dalton in Ref. 1 would be a very powerful method for calculation of potential flow at nonzero angle of attack past an arbitrary axisymmetric body.

The second limitation to the comparisons presented in Ref. 1 lies in what must be an inadequate formulation of the higher-order (linear and parabolic) axial source methods utilized for the more complex body shape shown in Fig. 2 of Ref. 1. This body shape has a flat midsection and, hence, a discontinuity in curvature, from a finite value to zero, as one moves from the nose section to the cylindrical midsection or from the midsection to the tail. As shown in Refs. 3 and 4, accurate representation of the potential flow around such bodies is possible at least for the linear source distribution, both at zero and nonzero angle of attack, only if the usual continuity-of-source-strength requirement is eliminated for the two abutting axial-singularity elements at the nose-cylinder or cylinder-tail juncture. This source strength continuity condition was replaced by an additional control point located on the body surface near the juncture. Calculated surface pressure coefficient distributions were quite smooth and agreed well with experimental results at zero[3] and nonzero[4] angle of attack, as seen in Fig. 1 taken from Ref. 4. Comparisons are made with experimental results of Ref. 7. Solutions obtained using the piecewise linear source singularity for this class of axisymmetric bodies oscillated wildly when the source strength was forced to remain continuous across a jump in body curvature. Also, the source strength distributions obtained for the accurate flowfield representation (no continuity requirement at the juncture) displayed a discontinuous jump at the juncture from positive (source) within the nose to negative (sink) within the cylinder. This jump in strength was apparently necessary to turn the flow to be tangential to the cylindrical portion of the body just downstream of the juncture. It is expected that elimination of these continuity requirements for the linear axial source method used for the second example in Ref. 1 should lead to significant improvements in accuracy. However, it must be noted that these comments support the basic conclusion of D'Sa and Dalton, that axial-singularity methods are less reliable than a surface-singularity formulation, since accurate axial-singularity results appear to require relatively higher numerical precision and, at times, insight into the appropriate choice of element size distribution.

References

[1]D'Sa, J. M. and Dalton, C., "Body of Revolution Comparisons for Axial- and Surface-Singularity Distributions," *Journal of Aircraft*, Vol. 23, Aug. 1986, pp. 669–672.

[2]Hess, J. L., "The Unsuitability of Ellipsoids as Test Cases for Line-Source Methods," *Journal of Aircraft*, Vol. 22, April 1985, pp. 346–347.

[3]Kuhlman, J. M. and Shu, J.-Y., "Potential Flow Past Axisymmetric Bodies at Angle of Attack," *Journal of Aircraft*, Vol. 21, March 1984, pp. 218–220.

[4]Shu, J.-Y. and Kuhlman, J. M., "Calculation of Potential Flow Past Non-Lifting Bodies at Angle of Attack Using Axial and Surface Singularity Methods," NASA CR-166058, Feb. 1983.

[5]Moran, J. P., "Line Source Distributions and Slender-Body Theory," *Journal of Fluid Mechanics*, Vol. 17, Pt. 2, 1963, pp. 285–304.

[6]Karamcheti, K., *Principles of Ideal-Fluid Aerodynamics*, Wiley, New York, 1966, pp. 342, 343.

[7]Campbell, I. J. and Lewis, R. G., "Pressure Distribution: Axially Symmetric Bodies in Oblique Flow," Aeronautical Research Council CP-213, 1955.

Reply by Authors to J. M. Kuhlman

Joseph M. D'Sa* and Charles Dalton†
University of Houston, Houston Texas

WE have reservations about several issues raised by J. M. Kuhlman in his Comment on our paper.[1] First, Kuhlman is reminded that the "exact" representation of an ellipsoid of revolution by linear variation of sources between the foci is valid only under a certain "simplifying condition."[2] Even though the ellipsoid of revolution may represent an "exact solution" when the foci represent the element endpoints, this situation does not apply to our calculations because the foci did *not* represent the element endpoints. We used empirically determined distances of 2 and 98% of the body length to begin and end the element distributions. Not using the foci as element endpoints generates a numerical solution, which can represent a test case for comparison to the exact solution. In addition, it is probably true that Kuhlman's CDC calculations contributed significantly to the small errors that he reports.

The authors are puzzled by what Kuhlman means when he suggests the superposition of his line-doublet singularity distribution with a surface-singularity method to represent a body at angle of attack. When a surface-singularity method is used, it is not necessary to augment it to represent an angle-of-attack flow.

Second, Kuhlman suggests representing complex body shapes by imposing specific conditions that are far from simple on the distribution of the axial sources. Even though Kuhlman used the abutting singularity elements at the nose-cylinder or tail-cylinder junctures, there is no mathematical basis for choosing any particular adjacent elements to effect this condition. The question then becomes a matter of where the discontinuity in source strength is allowed to occur in the source distribution. This condition seems to be based purely on computational experience, especially since the axial-singularity solution is not unique.

The authors are also puzzled by Kuhlman's comment concerning a curvature discontinuity in our second example. We do not see this body as having a discontinuous curvature.

In conclusion, we understand that Kuhlman is attempting to improve the accuracy of the axial-singularity method by placing restrictions on how the axial singularities are located and distributed. Kuhlman suggests improved numerical precision, and we certainly agree. Kuhlman is reminded that simplicity of formulation and application is the strength of the axial-singularity method. With the modifications suggested by Kuhlman, the method loses some of its simplicity in formulation and becomes more of an art in application. The second test case of D'Sa and Dalton, the "complex" body shape, was the true application we sought to make. The more complex the body, the more innovative one needs to become to implement the axial-singularity method. In essence, the more complex the body, the more the method loses its simplicity. This tends to lead the user back to the surface-singularity method, which we observed to be the superior method.

References

[1]D'Sa, J. M. and Dalton, C., "Body of Revolution Comparisons for Axial- and Surface-Singularity Distributions," *Journal of Aircraft*, Vol. 23, Aug. 1986, pp. 669–672.

[2]Durand, W.F., *Aerodynamic Theory*, Dover, New York, 1963, pp. 283–285.

Received Oct. 22, 1986; revision received Jan. 16, 1987.

*Graduate Assistant, Mechanical Engineering.
†Associate Dean of Engineering.

Comment on "A Ring-Vortex Downburst Model for Flight Simulation"

B. Etkin*
University of Toronto, Toronto, Ontario, Canada

IN Ref. 1, Ivan has improved on the numerical calculation method given in Ref. 2 for computing the velocity at an arbitrary point in a simulated downburst. The essence of his method is to use the closed-form solution for the stream function of a ring vortex, which is exactly equivalent to that of the

Received May 19, 1986; revision received June 20, 1986.

*University Professor Emeritus, Institute for Aerospace Studies.